Relativity and Causality. From the Age of Enlightenment to the Message of 9/2011

Zafar Turakulov

Ulugh Beg Astronomical Institute of the Uzbek Academy of Sciences
E-mail address: tzafar@astrin.uz

Abstract.

The role of so-called relativistic causality principle in physics is traced from the Enlightenment Epoch, when it was introduced for the first time by J. D'Alembert, to 21 of September, 2011, when measurements of speed of neutrino completed by laboratories in CERN and OPERA exposed an unexpected result. Motion with superluminal speed was the first experimental evidence of the principle breakdown which changed the status of the principle, though in theory it was violated many times before. In this book we analyze the most considerable cases of the relativistic causality principle breakdown starting with the electromagnetic field of a particle which was completely neutral till some moment of time and then polarized such that its electric or magnetic moment changed from zero to a finite value. It is shown that the principle was violated also in the phenomenon of emissions of radiation by a moving charge, in the field theory on classical level in the framework of Chew-Low model of pion-nucleon interactions and even in celestial mechanics. An experimental observation of superluminal speed pushes forward new demands. First, it must be explained why neutrino moves so. Interactions, this particle is involved in, are considered and conclusion is made that neutrino is involved in a specific interaction with gravitational field of the Earth which is possible only if this particle carries a non-zero gravimagnetic charge. An equation of motion is presented which allows neutrino to move faster than light.

Contents

Introduction

1. Revolutions in society and physics

During the whole of *XX* century everything was completely clear about relativistic kinematics of particles and its connection with the notion of causality. It was known that only massless particles can travel with speed of light and no other value of speed is possible for them. Other particles move slower and need infinite energy to reach this speed. A particle which can be emitted and absorbed by other objects cannot travel faster than light because this would break causality. An assumption of existence of super-luminal particles allows to perform apparently impossible situations in which an event cancels itself after it already happened (a couple of examples will be considered in the next chapter). Therefore impossibility of super-luminal speeds was identified with causality itself and called "the relativistic causality principle" whereas the idea itself that something can move faster than light seemed to be something absolutely impossible even in particle physics. The more unexpected the message from CERN and OPERA [**1**] about speed of neutrino, which exceeds the speed of light, was.

Just before this message was issued, everything seemed to be completely clear about the future of fundamental physics. All kinds of interactions are already known, the corresponding theories have been composed, future experiments can only confirm them. These theories are based on fundamental principles, one of which is the relativistic causality principle. The message undermined this belief. It turned out that physics of *XX* century will not be exactly what it was expected to be. It looks like that the CERN-OPERA experiment is a landmark to separate the centuries.

Each century has certain starting and ending dates found in a calendar. The notions like "*XX* century" or "*XXI* century " have rather historical meaning which has nothing to do with calendar dates. In his book *On History* E. Hobsbawm [**2**] relates starting and ending points of some centuries to certain historical events. For example, the *XIX* century lasted from the French Bourgeois Revolution to decay of the last empires in Europe in the World War *I*, thus, lasted more than hundred years. The next *XX* century was short not only because of late beginning but also due to early end which is crush of Soviet Union. His idea can be used another way. Taking the dates from history of physics yields the following division. The end of the *XVII* century can be identified with publication of the I. Newton's "Principia", the end of the *XVIII* – with discovery of the first relativistically-invariant equation by J. D'Alembert and of the *XIX* with discovery of M. Planck's constant, or, maybe, of atomic nuclei by E. Rutherford. Thus, the *XX* century started in 1900 or 1904 and finished with closing the Superconducting Super Collider project that was the end of the high energy physics.

Beginning of each new century brought a new way of thinking or non-thinking and can be characterized by it. So, *XVIII* century can be characterized as the age of analysis because this mathematical science underlies all theoretical achievements of physics of the time. Within the next *XIX* century many important fields of knowledge were developed, but the most active development was seen in mathematical physics, therefore, it can be called "the age of Laplacian". As for the *XX* century, it passed under ideology of physics of microcosm.

Many important discoveries have been made during each century, which contributed the forthcoming revolution in physics. The *XIX* century was is especially rich of theoretical achievements among which there are Lagrangian and Hamiltonian mechanics, Maxwell's electrodynamics and the whole of mathematical physics as it is found in standard textbooks, for example, [**3**]. These achievements contributed mathematical sciences like differential geometry and functional analysis. However, few last decades of the century are known mostly as period of rush development of practical applications of knowledge accumulated before. By the end of *XIX* century electricity and radio were in use, as well as many other things appeared thanks to development of physics as a whole. In *XX* century applied physics became the mainstream in this science.

2. Ideology of the second half of *XX* century

At the very beginning of the second half of the *XX* century physicists were put under strong control and a new generation came which was completely different. The main feature of the new generation is that basic ideas of physics were never interesting to them, instead, they were trained to study only formal aspects of a new theory which allow them to calculate anything and start calculations as soon as possible. Anything could be fed them in the guise of a physical theory because they are responsible only for results of calculations. Meaning of physical theories was completely lost. A typical theorist of the time believed that all problems which could be solved analytically, are already solved and future of physics is completely digital. Another typical belief of a theorist of this formation reads that there are no nonlinear problems in physics, because thanks quantum theory any physical system can be quantized, hence, possesses a linear space of states. As such, everything but functional analysis became insignificant because any linear system is equivalent to a linear functional space.

It seemed absolutely clear what theorists must do in the future. Their only task was to enumerate all possible field Lagrangians and calculate all possible Feynman diagrams. As everything reduces to calculation, there is no question, what is space, time and space-time. No space or space-time longer exists, they have been replaced by lattices, and field is nothing but an integer specified in each lattice point. Calculation is all and all the rest is a kind of strange activity of physicists which keep abreast of the times.

It was triumph of ideology of *XX* century. New times came when theorists did not longer need to try to imagine or understand anything[1]. Reasoning became unnecessary at all. It was replaced with blind and unconscious calculations which provide numbers as the only reality and the only goal of any activity of any theorist. Theorists of the time believed that one can understand what he cannot imagine because to understand it to calculate. Search for analytical solutions was actually banned. Further search for analytical solutions is nonsense because since, in any case, the final goal are numbers, nowadays we have much more effective ways to reach it without solving any problem and, maybe, thinking at all.

Contrary to all this, search for analytical solutions requires reasoning. First of all, before starting such a work one must think well, whether he is able to find any. If it is not given to him, he can waste the whole of his life, so it is better not even to try and do something else. Calculations are the best choice for those unable to solve problems, because as a rule, calculation guarantees some result. Besides, they require no thinking, one

[1]L. D. Landau created an ideology in theoretical physics, due to which one can understand what he cannot imagine

must only to be careful about some formal grammar when collecting similar terms, removing brackets and so on. Presently some computer programs accomplish this work better than human.

A giant amount of calculations have been completed during decades since Feynman's techniques appeared, and it looks like, almost none of them have ever been used. Nobody can tell now, whether their authors ever believed the results. Contrary to it, almost every analytical solution has an everlasting value for theoretical physics and each of them costs, maybe much more than all numerical results together. Nevertheless, new ideology of *XX* century in theoretical physics apparently tried to destroy everything that could lead to obtaining analytical results.

Gradually, this ideology has formed a new community of theoreticians, whose members successfully adapted themselves to it. None of them even asked whether infinite values of mass and charge of an electron which calculations yield, mean that something is basically wrong in the theory. Everything was simple for them: if a result is infinite, just subtract infinity from it, get any result they want and call this procedure "renormalization". Renormalize all theories, including non-renormalizable ones, journals will publish this, because, after all, nobody cares, whether or not journal publications convey any meaning. Due to the ruling ideology, those unable to accept all this, have no abilities to be physicists. So, the ideology of theoretical physics of *XX* century has successfully formed a totalitarian society. Its members believe today in what is said today and will believe tomorrow in what will be said tomorrow. Quite recently they believed that the space-time is 4-dimensional and soon after that they already believed that it has 2, 10, 11 and 26 dimensions. It does not matter for them, whether the space-time exists at all, their task is to calculate anything. It does not matter, whether their calculations are meaningful or not, because if someone is doing what others do, he is right.

3. Unfulfilled promises

Creation of quantum mechanics several decades prior to this event is known to change world-view of physicists. Since this theory is valid only in microcosm, such a change means that physicists of the early *XX* century decided to restrict their world with atomic and sub-atomic scales. Classical theories were divided into two classes, one of which was contains theories to be quantized, and another which does those which do not exist on microscopic level. The earlier were to undergo reduction which should turn them into introductions into the corresponding quantum theories and the latter were to be excluded from fundamental physics. For example, mechanics of mass point turned into an introduction to quantum mechanics whereas fluid and gas dynamics became a branch of mathematics. Thus, ideology of *XX* century physics defined frontiers of this science as something related only to microcosm whereas everything related to larger scales was qualified as something obsolete, old-fashioned and, after all, not interesting from fundamental point of view.

However, quantum mechanics was just a visible part of the new power in physics. Clear outlines of a power over quantum mechanics were seen beyond physics. This power revealed in foundations of this theory, which read that any quantum system represented by its space of states. Usually this space reveals in the form of an orthonormal set of particular solutions of the Schrodinger equation for this system, called "pure states". Thus, after all, any physical system is identified with a linear functional space, so, the whole of fundamental physics of *XX* century was a branch of linear functional analysis. Physicists really believed that thanks to quantum mechanics all non-linear problems gone and a bright future is ahead in which only straightforward calculations are needed. But the name of the game was atomic nuclei. This ideology rose a hope that the most actual objects of the after-war period which are nuclei, will be described this way quite soon. This did not come true and physicists had to change the goal. By the time, rules also changed and to go on, physicists had to promise always something great.

They supposed that elementary particles encapsulate a more powerful source of energy than nuclei, and made elementary particles physics and field theory more actual than nuclear physics. First of all, they had to find a structure of nucleons that requires to collide particles at higher energies than those reached before. They were able to claim that something important will be found at higher energies, and those who decides whether or not to continue this, had to believe them for decades. However, it turned out soon that there is nothing more to promise on this level. The new promise was to compose the general S-matrix of all possible reactions from the experimental data. If this work is completed, no field theory is longer needed, no more equations will ever appear and the only task remains, to expand this object as an analytical function of many complex variables.

After that giving more and more promises became the most important activity and a kind of struggle for support. The new idea was to unite fermions and bosons by extending the space-time with spinor dimensions. In fact, this can be done only under an additional condition that the space is flat, but an ordinary theorist cannot guess of this. The point is that he grew in flat space endowed with standard Cartesian coordinates and always believed that geometry is nothing but an introduction to linear algebra, therefore such an extension seems quite natural to him. The idea of adding spinor dimensions to the space-time allowed some theorists to start a chain of great promises. First, they promised to remove all divergences from the field theory. Surprisingly enough that those who convinced others that infinite renormalizations in quantum field theory encapsulates the deepest understanding of physics, now started to promise to eliminate this delusion, but nobody asked them, where is the truth. Besides, they predicted discovery of superpartners to all known particles. The next promise was to quantize gravity. When they saw that these promises will never be fulfilled, they claimed that now everything is clear, the notion of particle must be replaced with that of string. The next great idea was that the space-time must have dimension equal to 2, 10, 11 or 26 because otherwise theorists cannot complete their calculations. It was seen in 80-s that new tactics of theorists is to promise something new before the time comes to fulfill promises given before. It was an unmistakable sign that the ideology which was reigning over physics for decades, was exhausted long ago. Community of physicists became completely totalitarian. A tiny opposition among theorists which worked on general relativity was not a significant force in it. Therefore the age of microscopic physics finished without revolution and its end was predetermined by the end of financial support of high energy physics. This fact signifies that high energy physics was not a field of genuine science, because genuine science survives not only without any support but even under oppression.

4. The end of ideology of *XX* century

In its last stage ideology of *XX* century made blind calculation the only possible activity of theorists. However, calculations yield only cyphers. All the rest comes from reasoning. Early 50-s brought the most interesting achievements of the time, namely, spatial parity violation in weak interactions, so-called NUT solution of the Einstein's equation, discovery of the Papapetrou equations and the idea of local symmetries. The NUT solution signified that general relativity admits existence of gravimagnetic charge as a source of the field, A. Papapetrou's analysis of equations of motion showed that world lines of spinning particles are not geodesics and the idea of local symmetry led to the theory of non-Abelian gauge fields, which presently underlies the theory of fundamental interactions.

Two of these discoveries exposed violation of one of basic principles of general relativity, which reads that the world line of a freely moving particle is a geodesic in the space-time. First, it turned out that the world line of a spinning particle is a curve satisfying the Papapetrou equation which differs from the geodesic equation. Second, the NUT solution of the Einstein's equation confirms possibility of existence of gravimagnetic charge. If it exists, there exist elementary particles to carry it and, as seen from non-relativistic considerations, their world lines are not geodesics. Moreover, this solutions actually leads to breach of another fundamental principle, due to which the space-time is a Riemann manifold [**4**]. However, the main activities were observed in another direction.

Fast progress of technologies of that time allowed to build huge particle accelerators and, thereby, so-called big physics. Big physics is an activity of big societies serving big machines. A machine and the attached society together constitute an integral whole that is possible only under a certain condition: every member of the society must accept a certain ideology. This is obligatory first of all, for theorists, because they play the same role in this society as the ruling party in any totalitarian society, thus, are responsible for stability of the power. Their main duty is to confirm the ideology, such as superstring theory and it is unknown what they really think about physics and whether they think anything at all. They apparently have nothing in common with scientists of passed centuries.

Everything finished unexpectedly. Dismissal of the Soviet Union did not seem to be something fatal for high energy physics and big structures, but one year later the greatest project in high energy physics was canceled. Suddenly everything called "fundamental physics" became unnecessary. Not only new knowledge about "the depth of the matter" but also big accelerators, the whole of ideology, and people which believed in it.

Just few months back everything seemed to be clear. The whole of science depends on physics, the whole of physics depends on its foundations. Fundamental research is the same as explorations of the matter on subatomic, sub-nuclear and so on levels. Only high energy physics approaches these levels of knowledge. All the rest physics is based on obsolete concepts of passed centuries and is much closer to enginery than to fundamental science. Suddenly some new reality revealed which tells that the world does not longer needs all this but it does not tell in which point the ideology which reigned for a century, is broken. Physicists had to find this point themselves.

They have got a good excuse to recall what their fundamental research looked like. Formally, their research work was always aimed to completing promises they gave before. For some reasons, these promises have never been completed. Each promise was based on their beliefs and seemed to be quite accomplishable. In the next section we discuss the beliefs which afterwards turned into ideologies and scientific programs and after all, failed.

5. The great leap

The new age came, but unlike previous ages, this one was not brought by any new ideas. Physicists, which grew in the totalitarian society of the age of microscopic physics, were accustomed to believe that only financial support defines what does and what does not exist in the nature, whereas no ideas make any sense. It was known only that physics of the new age will not be subject to the ideology of microcosm. In this state fundamental physics became an easy prey of external forces like TV, marketing companies and was drifting towards turning into a kind of show for curious people. The most successive scientific report of the time consists of 90% of advertising of new technologies, 9% of reminiscences and reverences, 0.9% of referenced data and all the rest was exposition of generally-known ideas.

Physicists foresaw these changes well in advance and tried first to defend the value of micro-physics by setting stories about soon end of oil and gas resources and necessity of rush development of nuclear technologies. This did not help and then it was found that the Earth is placed in a cloud of asteroids many of which can bring the end of the world. It was the last chance and as physicists tried to take it, they already agreed that the age of micro-physics is over. As such, no more huge financial support will be offered to it and the community of those worked in the field must be radically reduced. And finally, as majority of fundamental physicists have to find a new job somewhere in business, no asteroids were longer needed and they disappeared.

The first step made by fundamental physics in the very beginning of the new age, was a great leap from the smallest to the largest scales. Suddenly this science known before as a study of the depth of the matter turned into the study of formation of the large scale structure of the Universe. No fundamental physics is found in the interior of this range. In fact, behavior of physicists at this period showed that physics itself was not interesting to them, they were only people seeking for any job and agreed to be physicists to get it. They never cared of what do they believe in, what they are doing, and for what purpose. Many of them changed physics to trade and explained others that a good physicist is able to find job anywhere. As for physics, it became a kind of show and telescope became a new show star.

The most actual task of the high energy physics was now to prove that it is still needed. Paradoxically, but the main difficulty was that there was nobody whom this proof would be addressed. Community of involved physicists did not need any proofs, whereas free-thinking physicists were successfully oppressed. The only way out was to shock the world as a whole. To detect a speed greater than the speed of light c. Some neutrino-astrophysical observations have already exposed signs that neutrino has speed slightly greater than c, so it remains to design an experiment in which this phenomenon can be detected under terrestrial conditions.

6. The speed of neutrino

The best scientific show was performed by experts on high energy physics in CERN and OPERA laboratories. They have measured the speed of neutrino produced by charged mesons decay in CERN, Geneva, and detected by the OPERA laboratory in Gran Sasso, 731 km away from it. The speed measured slightly exceeds the speed of light. Before this result was announced, its authors thoroughly checked everything and accumulated statistics until it exceeded 16 000 events. The results were published only when the authors were absolutely sure that it is meaningless to look for possible mistakes. It took about three years before on 9/11 (September of 2011) the e-print [**1**] was published.

Majority of theorists were taken by surprise. The time when they felt sure that they understand physics, came to the end. Though they did understand that it is meaningless to look for mistakes in experimentalists' work and they never did so before, they either expressed their doubts or repeated that this is impossible. All this had no prospects and an explanation of the results should be found. They could try all ideas known to them and find that all these ideas only confirm impossibility of super-luminal motion.

Now, to get their sureness back, they had to recall and analyze everything. Statements that the causality principle is violated, appeared from time to time in the past, and now they all were to be studied thoroughly. All questions unanswered before and accumulated in physics, were to be answered now. Evidently, it is not a task which members of a totalitarian society, whose activity was reduced to blind calculations for decades, can accomplish. They found themselves in a situation when their calculating skills could not help. To accomplish the task they had now, they were to do things they never did, namely, to think, imagine, understand and be responsible for their conclusions. The main question, how a super-luminal motion can be possible, is to be answered in the framework of full-valued theory in which space-time is not extended with non-existing dimensions and particles are not superstrings. Hardly it is reasonable to expect that it will be answered by those who only confirmed ideology of totalitarian society and calculated what is said. The answer will be given below. That is what this book is about.

7. Brief contents of this book

In the Chapter 1 we discuss origin, meaning and usage of the relativistic causality principle. We start with the notion of principle and its place among other statements found in exact knowledge, which are definitions, theorems and others. After that we analyze inter-relations between the principle and the D'Alembert's method of characteristics and show that relativistic causality principle follows from properties of the D'Alembert equation. Then we pass to classical electrodynamics and show that this principle cannot be obtained from properties of Maxwell equations the same way. Moreover, we prove that there exists solution of the Maxwell equations for the field of an instantly polarized particle which breaks the relativistic causality principle. Hence, solution of Maxwell equations with given initial and boundary conditions is not unique and one can choose between those obeying and those violating the principle. In other words, relativistic causality principle is to be used the a selection rule after the most general solution is obtained. After that we return to the notion of causality and show that relativistic causality principle is sufficient but not necessary condition of causality itself. It can be weakened because under certain conditions existence of super-luminal particles alone cannot break causality.

In the Chapter 2 we consider the problem of instantaneous polarization of a neutral particle as a process in which compliance or violation of the causality principle can easily be demonstrated as soon as the corresponding solution of the Maxwell's equations are obtained. First, we criticize the traditional approach to Maxwell equations based on the method of Green functions in the case of the problem of the field of a moving charge. For this end we formulate two principles which actually underlie this approach and show how they were used, show how do they yield erroneous results and make a conclusion that the only opportunity to find the desired field is to solve Maxwell equations by the method of variables separation. In the rest part of the chapter we build an exact solution which manifestly obeys the causality principle. For this end we construct a special coordinate system which covers only interior of the light cone out from the space-time point in which the particle was polarized and solve the equations by the method of variables separation.

In the Chapter 3 we try to introduce retardation of gravitational force between bodies of into celestial mechanics, assuming that this force obeys the causality principle. First of all, we find that retardation of force violates conservation laws such a way that none of integrals of motion known in the two body problem can be derived from the equations of motion with retarding force. Moreover, these equations expose monotonic growth of values which are normally used as integrals of motion, particularly, energy and angular momentum of the system. Under these conditions even the notion of center of mass of the system is meaningless, however, the system has a certain approximate center of mass, which rests in an inertial frame, if retardation is neglected. Though equations of motion are too difficult to be solved analytically, some calculations can well be made, which allow us to estimate expansion of the lunar orbit. Two possible versions of retardation of the force provide two distinct results, exceeding observed data by four and two orders respectively.

In the Chapter 4 we derive equations of motion of a massless particle carrying gravimagnetic charge. Before doing this, we analyze the fact of super-luminal motion exposed by neutrino in the CERN-OPERA experiment and conclude that the only explanation of the phenomenon discovered, is that neutrino carries gravimagnetic charge. However, there are more subjects to be discussed. First, it should be explained, whether causality was broken by this phenomenon. We found that though the relativistic causality principle is violated, causality itself was not, thus the commonplace relativistic causality principle is a sufficient, but not necessary condition of causality. Therefore, we term it "the strong causality principle" and propose a "weak causality principle" due to which neutrino can travel faster than light, but its speed in the opposite direction is smaller than c. We explain such a dependence of speed of neutrino on direction by its negative parity which at the same time, signifies that this particle carries gravimagnetic charge. Then we derive equations of motion of a massless particle carrying gravimagnetic charge from the Papapetrou equation using the fact that spin is nothing but a gravimagnetic dipole. Finally, we propose an approximation which allows us to obtain some qualitative results which explain the result of the CERN-OPERA experiment.

In the Chapter 5 we obtain qualitative results from the approximated form of the equations of motion derived in the previous chapter. For this end, first, we show that in the non-relativistic limit our equations of motion coincide with the non-relativistic equation which underlies the notion of gravimagnetic charge. This fact signifies that our relativistic equations of motion correspond to dynamics of particles carrying gravimagnetic charge and can be applied a massless particle. Analysis of these equations shows that only a part of the Riemann tensor which contains angular velocity of the Earth's rotation, contribute the desired deviation from a geodesic. To obtain it, we construct an approximate solution of the Einstein equation for the field of rotating uniform ball and a geodesic flow which contains world lines of neutrinos used in the experiment. We obtain the explicit form of approximated equations of motion of a massless particle carrying gravimagnetic charge and show that a non-zero solution exists. Qualitative analysis of the equations shows that their solutions describe super-luminal motion in one direction and sub-luminal motion in the opposite direction.

This result allows us to predict that a similar experiment with neutrinos traveling from CERN to the north, say, Hamburg, will show that neutrinos traveling in this direction have speed smaller than that of light.

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CHAPTER 1

Space, time and causality

1. An offer which one cannot reject

Physics is often being represented as a science based mainly on knowledge obtained from experiments. However, any attempt to restore the whole of physics from these foundations fails and exposes a gap between experimental results and understanding physical phenomena. Practically, this gap is filled with ideas, conjectures and expectations, which finally take the form of well formulated principles. Each principle is to be accepted by everyone without any justification, only in believe that afterwards it will be justified by the whole pattern of physical world.

The causality principle is somewhat different. It only claims that impossible events cannot happen. No event can affect its past and, particularly, cancel itself. In theory of relativity this principle was changed such a way that no event cannot affect not only the past, but also anything contemporary with it. In other words, the relativistic causality principle allows any event affect only events which belong to the cone of the future out from it. Hence, there is a difference between causality in common sense and the relativistic causality principle. Indeed, the earlier cannot be rejected, whereas the latter needs some justification.

During the whole of *XX* century the relativistic causality principle was used in all theoretical considerations and remained in full agreement with experimental data. Everything changed when the speed of neutrino was measured in CERN and OPERA. The first reaction to the message from there was a thought that if everything is so, theory of relativity must be replaced with something new. In fact, it is not so, because only the relativistic causality principle was broken, whereas the theory of relativity remains in force. In fact, the principle neither underlies relativity nor follows from is anyhow.

Breach of a principle, if it occurs, rises many questions, one of which is, why this particular principle was accepted and what its breach means. One one hand, none of principles were ever proved, otherwise they would be theorems and on the other hand, none of them is an axiom, otherwise it would be termed so. And nevertheless, everyone is obliged to accept them without any proof or other justification. Now it turns out that one of them is violated, so, one of questions which arise, is why everyone had to accept this one and why to accept others. Therefore in this chapter we start our considerations about the place and the role of principles in exact knowledge.

One of ideologies which reigns in mathematics describes the bright future in which the whole of exact knowledge turns into a unified axiomatic system. This system will be based on rigorous definitions and axioms, all the rest should be obtained from proofs of lemmas and theorems. The whole body of exact knowledge will take the form parodied in W. Blaschke's book "Kreis und Kugel" [**5**]:

Satz 1. *Ein plus ein ist Zwei.*
Beweis: Trivial.
Satz 2. *Zwei plus Eins ist Drei.*
Beweis: Seihen Satz 1.
...

Due to this ideology, each scientist should believe that the whole of exact knowledge can be built by continuing this construction and must be honored to contribute it, particularly by either "improving" or eliminating every consideration which lies beyond this system.

However, there exist one strange kind of statements called "principles". They apparently lie beyond any formal structure of this sort. Unlike definitions, they do not give names. Unlike axioms, they do not specify rules of the game. An axiom can be rejected or replaced by another axiom that means only that one theory if replaced by another. Unlike lemmas and theorems, they are never proved. It is absolutely unclear, why a principle must be accepted. Nevertheless, a principle is always an offer which nobody can reject.

A principle looks like a law which was discovered same way as physical laws have been discovered in experiments. An example of a typical principle is found in elementary geometry. The Cavallieri's principle reads that if two bodies of same height have equal areas of horizontal sections at each height, then their volumes are equal. A sample of two bodies which meet its conditions are outlined on the Fig.1. One of applications of this principle is known as a method for calculating the volume of a ball and

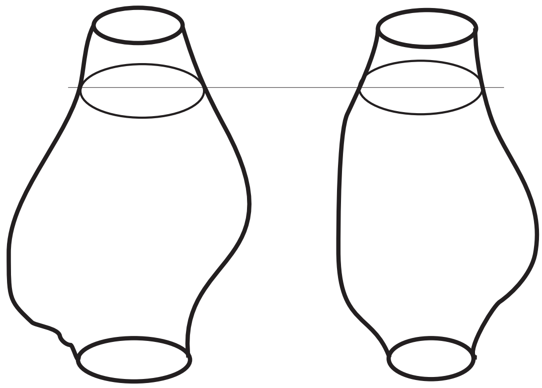

FIGURE 1. An illustration to the Cavallieri principle.

many other figures. No proofs of its statement exist, otherwise it would be called a theorem. One can accept it after imagining, for example, water dropping into two vessels satisfying the conditions, but this gives only an illustration which helps to understand it. Integration does not prove it because the principle itself underlies definition of this operation. Evidently, statements of this sort have no place in the bright future drawn by the axiomatic ideology. Due to it, if some of them are important, they should be turned into theorems by proving, otherwise they must be thrown out. So would be destiny of all principles including the so-called relativistic causality principle. In other words, if till now we have no any proof of this principle, hardly it will be proved in the future when no principles will exist, hence, it must be thrown away because in future science will not need them. Those who believes in this bright future, should not be concerned very much with discovery of faster than light neutrinos. Fortunately, axiomatic ideology is not the only possible view upon exact knowledge.

Below we employ three distinct relativistic causality principles which guarantee compliance of causality:

- The causality principle. No event can affect its past, particularly, cancel itself.
- The strong relativistic causality principle. No particle can move faster than light.
- The weak relativistic causality principle. No particle can move faster than light in two opposite directions.

The first of them is out of any discussion and must underlie any physical theory. The second is generally known as "the relativistic causality principle" and will be used until we need to replace it with a weaker condition that will happen only in the last chapters of this book. The generally known relativistic causality principle can be presented in various forms, for example, as a claim that once a particle was found inside some light cone, the whole of its world line since that moment on, lies wholly inside it. The third one sounds somewhat strange because usually speed of a massless particle does not depend on direction due to (local) isotropy of the space. So, if there are no distinguished directions in the space, this principle actually coincides with the strong one. The difference reveals in an anisotropic space. The space loses this property only in non-inertial frames, particularly, in rotating ones. It will be shown in the last chapters of this book that the strong causality principle can be violated in gravitational field of rotating matter and causality itself remain in force only by virtue of the weak causality principle.

Let us compare the strong causality principle with other principles encountered in physics. Some of them expose experience accumulated during millennia and claim that the space is uniform and isotropic and that a uniform motion of a frame of reference does not affect the laws of mechanics (Galileo's relativity principle). Another example is the action principle which is justified by coincidence of Euler-Lagrange equation with equations of motion. Unlike the earlier, the strong relativistic causality principle does not seem to expose any experience accumulated before. Unlike the latter, this principle provides no equations to be justified with. In further chapters other principles which have been used explicitly or implicitly in physics, will be discussed.

If a particle moves faster than light, chronological ordering of points of its world line is ambiguous and depends on the frame of reference. Regarding that CERN-OPERA experiment, one can find a frame of reference in which 16 000 neutrinos were emitted by detector in OPERA and met 16 000 mesons emitted in CERN just in time. Their collisions produced 16 000 muons detected there. All the rest events in this frame are ordered normally. If such a strange thing can happen in the frame of a muon, something like this is pretty possible in any other frame and should be observable. However, no observations of this sort have ever been reported. That is why results of this experiment seem to be strange. Perhaps, they signify that some strange things like this, are possible, but if someone observes something strange, he must admit that he moves too fast. Then, the relativistic causality principle is too strong and must be replaced with a weaker condition. It must be kept in mind that causality principle is our expectation based on the experience accumulated before and has nothing to do with definitions, axioms and theorems which may be proved.

2. D'Alembert's method of characteristics

J. Le Rond D'Alembert is a great thinker of the Age of Enlightenment. He is known mostly for his equation, but in fact, he knew many things which are believed to be discovered much later, particularly, Cauchy-Riemann conditions and Laplacian. In fact, he was first to add time as the fourth dimension and form a space-time with pseudo-Euclidean geometry known presently as the Minkowski space-time. In this section we start with his ideas as the origin of the relativistic causality principle.

His studies of small oscillations of a string led to the following equation:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\psi = 0,$$

where c stands for the speed of wave propagation along the string. This equation has particular solutions $\psi = \psi(x \pm ct)$ which have the form of an arbitrary function ψ depending on single variable $x \pm ct$. These two variables or the lines specified by them as $x \pm ct = \text{const}$ are called characteristics of the equation. The meaning of characteristics is seen from the fact that if initially ($t = 0$) the function ψ is non-zero only in a segment $[a, b]$, then each point of this segment acts as a source of the wave running away from it in both directions. As a result, at a moment of time t the function is non-zero only in a segment $[a - ct, b + ct]$ that would correspond to the relativistic causality principle if c was the speed of light.

The general wave equation for weak perturbations in an elastic matter has the form

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \triangle\right)\psi = 0. \tag{2.1}$$

D'Alembert has found characteristics of this equation in the form of cone $r = ct$, where r is spatial distance from the given point. Note that his characteristic cone coincides with the light cone if c is speed of light. The meaning of characteristics is the same. Each point of the space, in which ψ is non-zero, acts as a source, emitting a spherical wave propagating with speed c. The causality principle reveals in these solutions because the field ψ is non-zero only in the domain of the space-time, which lies inside the characteristic cone out from the point.

The operator appeared in the left-hand side of the D'Alembert equation (2.1) bears a strong resemblance to the Laplace operator and makes a hint about time as the fourth dimension. A deeper analysis would show what an equation and a space can have in common. They can have common symmetry group. Similarly to the Laplace equation whose form remains invariant under rotations of the space, the form of D'Alembert equation does not change under transformations of symmetry of the pseudo-Euclidean space. However, this hint alone does not justify the join because, first, the constant c in the equation depends on mechanical properties of the material and second, adding time as the fourth dimension implies existence of transformations of both time and spatial coordinates that cannot depend on properties of any material. The only kind of wave which does not need any material to propagate in, was light, but it was not known what equations it is governed with.

After J. C. Maxwell has discovered these equations, it turned out that, first, they differ from the D'Alembert equation and second, they have the same group of symmetries. The constant c, which in D'Alembert's equation characterizes properties of the matter in which the wave propagates, characterizes, first of all, vacuum in case of Maxwell equations. As a result, space and time were joined together, but one question remained. Is the relativistic causality principle in sense of J. D'Alembert, remain valid in case of Maxwell equations? It might remain valid if D'Alembert method of characteristics can be applied to them. The method has three different forms, for plane, cylindric and spherical waves and the question is, whether it is applicable in the spherical case. The problem is that spherical electromagnetic waves do not exist. It will be shown in the next chapter that neither the method of characteristics can be applied to, nor the relativistic causality principle follows from Maxwell equations.

3. Three kinds of particles

Mass is the number one characteristic of any particle. There exist three kinds of particles, namely, massive, which have non-zero masses, massless, which have zero mass, and tachyons. Almost all known particles have non-zero masses. Their velocities are always less than c because a particle with mass m and velocity v has energy

$$E = \frac{mc^2}{\sqrt{1-(v/c)^2}}.$$

In power accelerators they reach velocities very close to c while gaining high energies. As seen from this equation, to reach the speed of light a particle needs infinite energy, therefore, it cannot cross the luminal barrier that confirms the relativistic causality principle for all massive particles.

Massless particles are known to move with speed of light in any frame of reference disregards of their energies. Till now no physical factors were discussed which would change the square of a particle velocity. Therefore, c is believed to be the only possible value of speed a massless particle can possess, hence, massless particles have no rest state and cannot carry any electric charge which is always accompanied with some electromagnetic mass [**6**]. There exist two massless particles which are photon an neutrino. Below we outline interactions of these two particles with fields and a matter, which can affect their velocities. The notion of massless particle is an interesting object for our causality considerations because, on one hand, it is used in the formulation of the relativistic causality principle and, on the other hand, they do not need infinite energy to cross the luminal barrier.

The third kind of particles, so-called tachyons exist only hypothetically. A tachyon is characterized with a constant m which has dimension of mass and specifies the energy-speed relation as

$$E = \frac{mc^2}{\sqrt{(v/c)^2-1}},$$

so that a tachyon always moves faster than light. It needs an infinite amount of energy to cross the luminal barrier and slow down. A tachyon cannot be emitted and absorbed because these processes along with their super-luminal speed would violate the causality principle. Not a single tachyon is known; neutrino is not one because otherwise it would demonstrate much higher values of speed in the same experiment, besides, since the processes of their emission and absorbtion are known, they cannot be tachyons due to the causality principle.

Photon is a quant of electromagnetic field, so, everything is known about this particle. On quantum level all its interactions with a matter reduce to two possible actions which are absorption and emission of it by another object. On classical level, its interaction with a matter takes the form of reflection or refraction which would be more interesting for the present investigation because they change the particle velocity. However, the only impact of these phenomena on the photon speed is that in some media it is smaller than c. The only interaction with other boson fields is found in general relativity where photon is involved into the spin-gravitational interaction [**4**], [**7**], [**8**] due to which its world line obeys the Papapetrou equation

$$\frac{\mathrm{D}\dot{x}^i}{\mathrm{d}t} = \omega^{-1} R^i{}_{j\ kl} \dot{x}^j S^{kl}, \tag{3.1}$$

where ω stands for energy (Planck constant is put equal to unity), hence, is not a geodesic. As seen from this equation, the right-hand side specifies a vector of acceleration, which is strictly orthogonal to the vector of velocity. Therefore, this interaction does not affect the speed of photon, which remains equal to c. No other interactions of this particle with bosonic fields which would affect its world line, are known, therefore c is the only possible value of speed which it can have.

Unlike photon which serves as a carrier of an interaction, neutrino is a fermion, hence, represents a kind of field of matter. This particle is involved into two interactions. Like any matter, neutrino is affected by gravity, besides, it is an active participant of the weak interaction. As was shown above, the earlier does not affect its speed. The latter reveals only in microscopic scales, therefore, hardly it can make neutrino move faster than light on a distance of hundreds kilometers. However, our considerations of neutrino interactions are just began and we will return to in in the end of this book.

4. Space-like currents

The term "space-like" is one of three possible characteristics of a vector on the space-time. A vector can be space-like if it lies beyond the light cone with its origin as the vertex, a null-vector if it lies on the cone and time-like if it lies inside the cone. So are curves in the space-time. For example, world lines of all massive particles are time-like, and that of massless ones

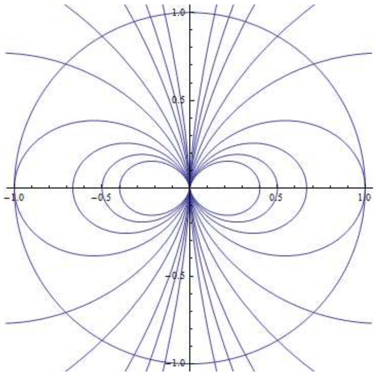

FIGURE 2. Lines of force of a point-like dipole.

are null-curves. Tachyons have space-like world lines and in general, this fact points to a possible causality breakdown.

If a particle carries an electric charge, it is associated with a vector of current density which is tangent to its world line, therefore, this vector is time-like. Space-like vectors of current density also exist in the nature. To see this, consider an electrically neutral wire which carries a direct current. The current density of such a wire is strictly space-like. Indeed, its spatial part is tangent to the wire whereas its time-like part is identically zero because, by conditions, the wire has no electric charge density. If the current depends on time as the Dirac's δ-function, i.e., exists only one moment of time (like an electric discharge) and is stretched in the space, it is strictly space-like. So would be current associated with a charged tachyon if it exists.

The notion of space-like current does not obligatory break the causality principle, but allows to describe an impossible phenomenon like disappearance of a charge without violating the charge conservation law. Indeed, consider a point-like charge which was at rest till a moment of time $t = 0$ and then disappeared. This is impossible due to the charge conservation law, but if we add a space-like current from the charge to infinity at the moment $t = 0$, its disappearance does not longer contradict the law and becomes formally possible unless the space-like current breaks the causality principle. Another possibility is that the charge blows up at the moment $t = 0$ and spreads over the space by strictly radial spatial current density. A similar impossible phenomenon is annihilation of a dipole under which the current lines are exactly the lines of force of the dipole shown on the Fig.2. Due to the well-known definition of the current of displacement $\vec{J}_d$

this current density is

$$\vec{J}_d = \frac{\partial \vec{E}}{\partial t}.$$

Evidently, this current density exists only one moment of time because the right-hand side of this equation contains the δ-function of t. This current density instantaneously transfers the charge from one pole of the dipole to another and thereby annihilates it. Note that exactly the same current of displacement describes the inverse phenomenon in which a dipole is created. In the next section we show that there exist solutions of the Maxwell equations for a dipole appeared or disappeared this way, which breaks the causality principle.

5. Polarization of a particle

Consider a completely neutral point particle which does not produce any electromagnetic field and whose state changes at the moment of time $t = 0$ such a way that its dipole moment becomes non-zero, thus, behaves like the well-known step function $\vartheta(t)$. This phenomenon is called polarization of the particle. Consider electromagnetic field of a particle polarized at the moment of time. It was shown in our book [**9**] that electromagnetic field of such a source cannot be obtained as the retarded potential and there is no other way than to solve the Maxwell's equations as they stand. This will be done in the next chapter and in this section we show that solution of the Maxwell equations with same boundary conditions is not unique and the causality principle must be used as an additional condition.

The field of an instantaneously polarized particle can be built by parts. Evidently, under $t < 0$ the strengths are zero and it remains to find the field under $t \geq 0$. So, it is natural to suppose that under $t > 0$ the field is equal to sum of two terms, one of which is stationary field of the dipole formed by the process, and another is unknown. Evidently, the unknown part has no source and describes radiation emitted by the particle at the moment of polarization. Therefore, the task is to find the unknown term.

By construction, the unknown strength satisfies the source-free Maxwell equations under $t > 0$, therefore, it can be represented by some solution of these equations multiplied by $\vartheta(t)$. As a result, the total strength contains this step function as a common factor. Since differentiation of this factor yields a δ-function singularity and this singularity appears as a current of displacement that is an extra source, it must be eliminated. This can be done by choosing the relevant boundary conditions for the source-free Maxwell equations, which require that the total strength is identically zero at $t = 0$. If the unknown term is obtained as a solution of the source-free Maxwell equations with these boundary conditions, therefore, the total strength is continuous and no extra sources appear at $t = 0$. Hence, at this moment of time the unknown term is equal to the strength of the dipole with opposite sign. Then under $t \geq 0$ we have a field given by a stationary dipole strength and a wave given by the corresponding solution of the boundary problem for source-free Maxwell equations. Unless something is wrong in this construction, this solution describes radiation emitted at the moment of polarization. However, it does not satisfy the relativistic causality principle.

The point is that the dipole field which plays the role of initial value of the radiation field is specified in each point of the space. Solution of the Maxwell equations with these initial values yield the strengths which appear everywhere in the space disregard of demands of the causality principle. On the other hand, we only supposed only that under $t > 0$ the field contains the dipole strength as one of summands, that does not seem to break the relativistic causality principle. Moreover, since the problem reduces to the initial value for the source-free Maxwell's equations, the solutions obtained this way are unique, so, there is no opportunity to take the causality principle into account. As a result, radiation appears everywhere in the space instantaneously in the particle rest frame. In any other frame radiation appears somewhere before the particle started producing any electromagnetic field that is impossible. In other words, this solution of the problem of the field of an instantaneously polarized particle apparently violates the causality principle. This fact signifies that unlike D'Alembert equation, Maxwell equations do not encapsulate the causality principle. So, if at the very beginning, instead of the D'Alembert equation physicists started with Maxwell equations, no causality principle would be discovered at all.

6. Causality and interaction

In quantum field theory, interactions between particles of matter is represented as an intermediate boson exchange, whereas on classical level another idea of interaction is in use. Due to it, each body produces a field with its mass or charge, which presents in each point of the space and therefore acts instantly at an arbitrary distance from the body. Thus, any particle along with its field constitutes an extended object which does not need to emit and absorb waves to interact with others. These two ideas of interaction exclude each other and to describe any interaction one needs to make a choice between them. It is convenient to reduce these two ideas to two principles only one of which can be accepted. Each of them affirms a kind of credo which everyone accepts without any justification.

- The principle of localized objects. Each particle interacts with others via emitting and absorbing waves. Therefore interaction has a certain speed of propagation, which coincides with the speed of propagation of its waves.
- The action at a distance principle. Each object and its field constitute an extended object which is an integral whole, therefore its field interacts with others instantaneously.

Both of them sound quite plausible and none can be proved or accepted as something evident. The earlier is in full agreement with the relativistic causality principle and underlies the ideology of XX century. The latter is apparently more relevant in general relativity where no localized objects exist and each object affects geometry of the whole of space-time. Gravitational interaction cannot be reduced to emitting and absorbing waves by point-like particles. Therefore the second principle is in force in general relativity, but it is unknown, whether it is compatible with the causality principle. As for the earlier one, it is unknown, whether it always provides satisfactory results, in other theories of interaction. These questions will be considered in forthcoming chapters below.

The principle of localized objects implies compliance the relativistic causality principle under certain conditions which require either that the theory of interactions used, is based on the theory of D'Alembert field or that all massless fields are equivalent in some sense. Namely, all field equations for massless fields encapsulate this principle by construction. However, it is not so. It will be shown below that the first realistic theory of interaction which is classical electrodynamics, does not contain it.

Unlike the D'Alembert equation, Maxwell equations do not imply causality, therefore there must be errors in considerations which underlie "derivation" of the principle from the Maxwell equations. Considerations of this kind are found in generally known books of R. P. Feynman [**10**] and

L. D. Landau [**11**]. These considerations are based mostly on replacing the original Maxwell equations with the D'Alembert equation that look more or less plausible only in Cartesian coordinates, though the idea alone that the system of 8 equations to which Maxwell equations can be reduced, is equivalent to the single D'Alembert equation, needed justification. Now it turns out that they are not equivalent because the latter implies causality whereas the earlier do not. The main object of these considerations is presented by plane waves whose phase and group velocities coincide with c. It is especially important that these authors never leave standard Cartesian coordinates in which some particular solutions of the Maxwell equations contain the factor $e^{\imath(\omega t-kz)}$ that gives the fixed phase velocity ω/k. Solving the same equation in the round cylinder coordinates yields solutions in which the phase velocity can be obtained from the factor $e^{\imath(\omega t-m\varphi)}$ and is equal to $\rho\omega/m$, thus, takes all possible values from 0 to ∞ depending on the value of the coordinate ρ. This example shows that in general, phase velocity of a wave and velocity of propagation of an interaction have nothing to do with each other. In any case, the causality principle does not follow from the Maxwell equations, therefore, solutions of these equations can well describe violations of this principle. In the next section we consider possible consequences of this fact.

7. Impossible phenomena

The causality principle protects one's thinking from unnecessary and wrong ideas about what can and what cannot happen in reality. In physics it's role is especially important, therefore there is no question, whether the principle can be violated. Since the principle is always in force, the question arises, what was happening in CERN and OPERA during about three years. An answer will be given in the end of this book, but now we need to consider many other things. It was always believed that abidance of everything to the principle is guaranteed by Maxwell equations and it turns out that in fact these equations have nothing to do with it. Consequently, there exist other guarantees that it will be always fulfilled. In the next chapter some of them will be considered.

It was noted above that in the CERN-OPERA experiment a strange situation was created that in the muon frame of reference neutrinos traveled northwards from Italy to Switzerland. This situation looks really strange, but it cannot be qualified as an absolutely impossible one. Probability of

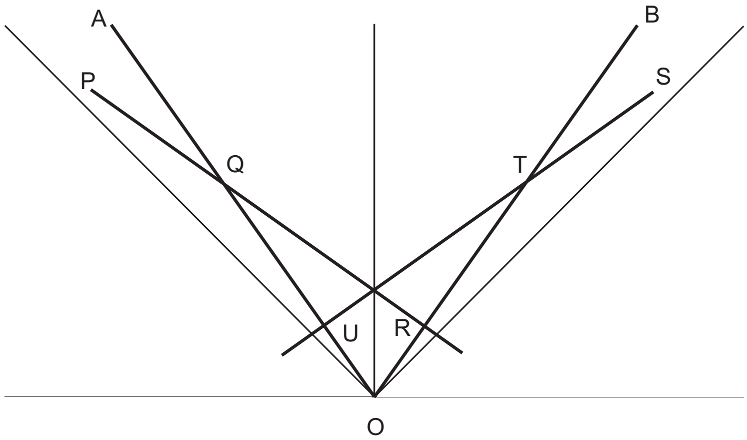

FIGURE 3. An impossible event.

emission of a neutrino by the detector in OPERA is almost zero, but not identically zero. It looks like that strange situations of this kind are possible, proviso that besides the muon's frame there also exists the laboratory frame in which the only strange thing observed is exceeding the speed of light by neutrino. So, now our immediate task is to find an example of an absolutely impossible phenomenon which is prohibited by the principle.

A point-like particle which can be polarized at some moment of time is an idealization of a really existing objects like a conducting loop which constitutes a magnetic dipole under a direct voltage. Polarization of such a loop produces actually the same field as a magnetized particle. Therefore, should solution composed this way be realistic, switching on a direct voltage onto such a loop would produce simultaneously a field of radiation in each point of the space. The difference between a loop and a particle is that unlike a particle, a loop can be switched on or off at any moment of time and theoretically their fields can be made arbitrarily strong. These properties of a loop and its field allow to perform an impossible phenomenon.

A performance needs two loops of this kind and a switching mechanism which works as follows. One of them can be switched on at any moment of time unless another is on. Another can be switched on only after the first one is switched on. Initially both of them are off. If they are in same frame of reference, switching on of the first one allows to switch on the second one so that finally both of them are on.

An impossible phenomenon occurs if they move fast in opposite directions. A scheme of such an experiment is shown on the Fig.3 where OA and

OB stand for world lines of the two facilities, the straight lines on which the point P, Q, R and S, T, U lie, show the spaces of their rest frames. Let the facility A can be switched on first and it is done at the point Q of its world line. Then, in its frame the radiation appears in each point of the space, in particular, in the point R of the world line of the second facility B. The second facility is free to wait for a while and switch on in the point T of its world line. This action entails emitting radiation in each point of the space in its frame at the same moment of time. This space crosses the world line of the facility A in the point U which precedes switching it on. Consequently, the facility A have been blocked at this moment and could not be switched on. As such, the facility B could not receive any signal from it and was not switched on. Then it did not emit any radiation which would block the switch on the facility A. Finally, we have an impossible situation in which, on one hand, A was switched on because B was off, and on another hand, if it is on, B switched on earlier in its frame so that A could not be switched on. This situation is absolutely impossible, consequently, polarization of a particle cannot entail emitting radiation in each point of the space at the same moment of time in its frame. This fact signifies that our approach to the field of a polarized particle exposed above is broken in some point an we need another analysis of this phenomenon. Evidently, another description of this experiment should be based on solutions of the Maxwell equations which obey the relativistic causality principle.

Now, let us return to the CERN-OPERA experiment and find conditions under which its result creates an impossible situation. Suppose that a meson (A on the Fig.3) has emitted a neutrino in the point Q of its world line. Let the neutrino traveled faster than light towards Hamburg (QR) where it collided another particle B which also moved northwards. Let after a while (RT) this collision creates another neutrino which travels back again faster than light. This neutrino is a space-like object in the frame of this particle (TU), therefore it collides the same meson before it decayed. After this collision the meson turns into another particle or particles. If this happens, no neutrino could be emitted in the point Q and instead of muon the CERN detector detects something else. In other words, in this case we have an impossible situation in which, on one hand, something else is detected, but on the other hand, it could not be detected. However, this does not mean that neutrino cannot move faster than light at all. This means only that neutrino cannot move faster than light in two opposite directions. If its speed southwards is greater than c, its speed northwards must be smaller than c and vice versa. If this condition is completed, no impossible situation can be implemented in experiments of this kind.

CHAPTER 2

Causality principle and Maxwell equations

1. The field of a moving charge: critiques of the traditional approach

In the previous chapter we have outlined a proof that there exists a solution of the Maxwell equations for the field of an instantaneously polarized particle, which breaks the causality principle. Such a solution is not acceptable for evident reasons, therefore there also exists a solution, which obeys the principle. Existence of two kinds of solutions signifies that, first, solution of Maxwell equations with given boundary and initial conditions is not unique and second, the causality principle is as important as these conditions. In this chapter we construct another solution of the same problem, which, unlike that one discussed above, manifestly obeys the relativistic causality principle. Before doing this we explain why we do not employ the method of Green's functions. For this end, we consider a related problem of the field of a moving charge, to which this method was applied and the result of this application is well-known. It will be shown that the method of Green's functions is not applicable not only to this particular problem, but to all problems of classical electrodynamics at all. After that we consider the field of a particle which was magnetized at some moment of time.

Almost all non-stationary problems in electrodynamics have been solved by the method of Green's functions. At the same time, this method is known mostly for its application to the problem of radiation from a moving charge. Therefore in this section we start with this problem to formulate principles which actually have been used as a kind of justification of application of the method of Green's functions to Maxwell equations, though these principles have never been formulated explicitly. An explicit formulation of the principle makes it possible to analyze critically usage of the method. In the next section we show that the method should not be applied to Maxwell equations and that these equations cannot be solved this way. Then we pass to that of the field of a polarized particle.

Traditionally, the field of a moving charge was studied due to radiation emitted. Therefore, this branch of classical electrodynamics is known mostly as the classical theory of radiation. The main goal of this theory is to describe radiation emitted by a moving charge and evaluate its main characteristics. From the very beginning, solution of this problem takes somewhat technical character which reduces the task to constructing only one particular solution of the Maxwell equation assumed to describe radiation emitted by a point-like charge drawing a certain world line in the space-time.

The standard approach is based on two main principles which, however, have never been formulated clearly. They read:

- The scalar and vector equivalence principle. Maxwell equations are equivalent to the D'Alembert equation, at least, in Cartesian coordinates.
- The straightforward reduction principle. The field of a moving charge can be built by combining the field of a rest charge, Lorentz transformation and retardation specified by the speed c of propagation of all electromagnetic perturbations.

These main principles of the classical theory of radiation need to be analyzed, but before doing this, we outline this theory as it is exposed in numerous texts and monographs.

If these principles are valid, the task divides into two. First, to compose the relevant solution of the D'Alembert equation and second, to transform it into the desired solution of the Maxwell's equations. Implementation of the earlier follows from the second principle which claims that an explicit form of the relevant D'Alembert field can be built by including pure retardation into a static field. In other words, if a stationary point-like source of magnitude q produces its scalar field of the form $f(r)$, a non-stationary one $q(t)$ does the field $q(t)f(r-ct)$. As it is done, it remains to transform this scalar retarded potential into the desired vector retarded potential produced by a non-stationary source like a moving charge. The second task was implemented with use of the first principle. Below we analyze how it was done.

Retarded potential is nothing but one of possible Green's functions integrated over the source. The Green's function itself is actually the field produced by a point-like source. There is no question of existence of point-like sources of a scalar field, but there is difference between scalar and non-scalar fields. The source of electromagnetic field is the vector of current density, which is constrained with the zero-divergence condition. This condition cannot be ignored because it is an expression of the charge conservation law. Therefore, the assumption of existence of point-like current densities which meet this condition, apparently was to be verified. Instead, physicists actually postulated that point-like currents which, however, do not break the charge conservation law, exist. This postulate also has never been formulated clearly, but was implicitly used as a formal justification of applicability of the method of Green's functions to classical electrodynamics. All the rest was just a technical problem and finally, the theory of radiation from a moving charge was composed. The method of retarded potentials was applied to many other problems, particularly, in antenna theory, as for the method of Green's functions, it was used as the main mathematical tool in quantum electrodynamics and afterwards, all the rest branches of quantum field theory were built on the same foundations.

Thus, the two principles formulated above, underlie a huge area of modern theoretical physics. This fact alone gives a more than sufficient reason to analyze the principles. Now, as they are formulated explicitly, this analysis is possible and is presented below. The strength of electromagnetic field has totally six components and Maxwell's equations can be represented in the form of a system of eight partial differential equations for these six unknowns. Besides, there exists a well-known transformation which reduces Maxwell's equations to a system of four partial differential equations for four components of the vector potential. Therefore, the first principle, which claims that this system is equivalent to the single D'Alembert equation, sounds somewhat strange. Under some special conditions these equations coincide with the D'Alembert equation. Relationship between Maxwell and D'Alembert equations has been studied in our book [**9**], where it was shown that this relationship is non-covariant and non-understanding this fact leads to erroneous results. In particular, the generally-accepted theory of dipole radiation built without understanding this fact and based on the theory of retarded potentials, is wrong. Consequently, the principle which claims any equivalence of Maxwell and D'Alembert equations, is erroneous. The second principle plays an auxiliary role and was used only as a justification of application of the method of Green's functions to

Maxwell's equations. However, as was shown in our book [**15**], this method cannot be applied to non-scalar equations for other reasons, hence, analysis of the second of the two principles formulated above is unnecessary. These considerations allow us to conclude that the traditional approach to the problem of the field of a moving charge is completely wrong and this problem still needs to be solved, that can only be done another way, namely, by solving the Maxwell equations as they stand without replacing them with a scalar equation.

2. Failure of application of the method of Green's functions

It is convenient to represent Green functions for the D'Alembert equation in a coordinate system with t being a Lorentzian time and r the distance between the two points. Apart from the retarded Green's function which is $G_{ret} = \delta(ct - r)/2r$ there exist also Green functions equal to $(c^2t^2 - r^2)^{-2}$ [**16**] that can be put non-zero inside or outside of the light cone. Evidently, G_{ret} is zero everywhere but the surface of the light cone and others are non-zero either inside the cone of the future, where $r < |ct|$, or beyond the cones, where $r > |ct|$. Application of these Green functions to the field of moving charge yields physically different results. The retarded Green function provides a vector potential in a point Q as if it was produced by the charge only in one point P of its world line which satisfies the condition $r = ct$, whereas the Green function which is non-zero inside the future light cone, describes potential produced by the whole of history preceding this point, thus, $r \leq ct$. Justification of the two main principles mentioned above is based on the fact that plane waves can be represented in terms of functions depending on single variable $z - ct$, where z is one of Cartesian coordinates. However, these considerations have nothing to do with the problem in question. It will be shown below that the whole of variety of Green's functions, is physically meaningful, therefore, all of them are legal and provide quite legal solutions of the D'Alembert equation.

Existence of more than one Green's functions shows that a particle does not need to possess any electric charge to emit electromagnetic waves. Indeed, let G_1 and G_2 be two distinct Green's functions, which provide two distinct formal solutions of the Maxwell's equations with one and the same right-hand side. Then, due to linearity of the equations, their difference is a source-free electromagnetic field, which describes radiation emitted by a neutral particle. Below we show that emission of electromagnetic radiation by a neutral particle, is in full agreement with the laws of classical electrodynamics.

A classical charged particle can be represented as a thin conducting spherical shell carrying electric charge. Similarly, a classical neutral particle can be represented as a charged capacitor carrying arbitrarily big voltage inside and zero total charge. If, for some reasons, the capacitor discharges, it emits radiation which, maybe, makes it accelerate. Therefore, there must be solutions of the Maxwell's equations to describe radiation from a neutral particle, so, it is not surprising that solutions obtained by the method of Green's functions confirm this fact. The main difference between radiation emitted by a moving charge and that of discharge of a capacitor is that the latter changes the total mass of the particle whereas the earlier leaves its mass unchanged. The total mass loss can be calculated by the Poynting theorem [**17**] which reads that time derivative of the total energy inside a surface is equal to the flux of the Poynting vector through this surface. This calculation was completed by many authors. A comprehensive exposition of calculations of the energy flux from a uniformly accelerated charge through the sphere of radius R with the charge as the center, is presented in the book [**17**], equations (14.14)-(14.19). The result shows that, first, the flux does not depend on R, and second, power of the radiation is proportional to square of acceleration (Larmor's formula). Since the flux does not depend on the sphere's radius, it remains valid in the limit of $R \to 0$. This fact signifies that the particle mass loss is the only possible source of the radiation emitted. Nevertheless, the result of this calculation is generally accepted as the expression for the power of radiation from a moving charge, that leads to the following paradox.

Due to the A. Einstein's equivalence principle, gravity is equivalent to non-inertiality of the frame of reference. Therefore, in a small enough neighborhood the field of a point-like charge suspended at rest in a static gravitational field, is exactly the same as the field of a uniformly accelerated charge. Consequently, due to the generally-accepted theory of radiation, a charge placed into a static gravitational field, acts as an everlasting source of free energy, breaking the energy conservation law. Alternatively, one can suppose that energy is conserved, but radiation spends the mass of the particle. If a charged particle loses its mass when radiating, masses of all charged particles should run away towards minus infinity, hence, this opportunity should be rejected.

Though this theory of radiation contradicts the energy conservation law and should not be accepted, it remains the only generally accepted theory of radiation for more than a century despite that in 1909 M. Born proved that the field of a uniformly accelerated charge contains no radiation at all [**18**]. His result was confirmed by A. Sommerfeld in 1915 [**18**], and much more recently we have published an exact solution of the Maxwell's equations obtained by the method of variables separation [**14**], which also confirms the M. Born's proof. All these facts expose consequences of applications of the method of Green's functions to Maxwell equations only in one particular case, namely, to the problem of the field produced by a moving charge. All of them signify that the method is useless in this particular case, but, as our considerations made in our books [**9**] and [**15**] show, it is useless in classical electrodynamics at all. Now we return to the problem of an instantaneously magnetized particle.

3. Polarization and causality

Polarization of a neutral particle is a process of creation of a source of electromagnetic field. The field produced by it, obeys Maxwell equation and the relativistic causality principle. To use this principle properly, we need first to fix the point on the particle's world line, in which it was polarized and second, to draw the light cone with this point as its vortex. The light cone divides the space-time into two parts. There is no question of the field inside the cone, and as for its exterior, the field there is identically zero due to the relativistic causality principle. Hence, solutions of the Maxwell equations to describe the field under consideration, are identically zero beyond the light cone. In other words, Maxwell equations are to be solved in the interior of the light cone. Alternatively, the equations could be solved in the half-space $t \geq 0$ and the field obtained this way would be in contradiction with the relativistic causality principle.

On non-relativistic level, time is one of independent variables but not one of coordinates. This circumstance creates two kinds of conditions, namely, initial and boundary ones. On relativistic level, time is one of coordinates, therefore all problems encountered there are of same kind and look like stationary ones. Unlike non-stationary problems, encountered on non-relativistic level, they contain only one kind of conditions, namely, boundary ones, though the boundary can be specified as a moment of time in various senses, say, as a 3-plane $t = 0$ or a light cone $ct = r$.

On non-relativistic level, there are numerous problems with source of a field defined on a surface or a curve as its limiting case. The problem of the field produced by such a source contains a boundary condition specified on the curve or surface. The source of electromagnetic field produced by a polarized particle is specified on the particle's world line, therefore, the corresponding boundary conditions are to be specified on this line. However, it turns out that the solution with this boundary condition is ambiguous, because one more boundary can be introduced, which can be the contemporary space $t = 0$ or the light cone $r = \pm ct$. The corresponding solutions are apparently different and a choice must be made between them. The choice can be made only on the basis of a certain idea of causality and choosing the relativistic causality principle entails adding a boundary condition on the light cone of the future $r = ct$.

In this chapter we construct the field of a particle which did not produce any electromagnetic field till some moment of time and then was polarized and started producing a field as a dipole. This construction will be completed under an additional requirement that the field does not break the relativistic causality principle. When solving the Maxwell equations we employ a coordinate system in which the causality principle can easily be expressed as a boundary condition.

There were many reasons to develop methods for solving equations of mathematical physics in various coordinate systems. Each coordinate system used for this ends is relevant in some cases and irrelevant in others, for example, Cartesian coordinates are good for describing plane waves, but less so for solving an equation with a point-like source or with boundary conditions specified on a surface of second order. In fact, variety of coordinate systems admitting complete separation of the main equations, provides variety of surfaces which can be used as boundaries in problems of this kind. The triaxial ellipsoid coordinate system [**3, 12**], used for solving ordinary problems of mathematical physics allows to specify boundary conditions on an arbitrary surface of second order, that provides complete solutions to broad variety of problems, encountered in practice. Usually, a boundary problem solves if the boundary is specified as a certain coordinate surface, say $u = u_0$ in a coordinate system $\{u, v, w\}$. In this particular case it is much easier to fit solutions of the main equations to boundary conditions, because they contain only one of independent variables that makes it possible to obtain complete analytical solutions of boundary-value problems.

Inclusion of time as the fourth dimension allows, in principle, to employ curved coordinate systems adapted to initial conditions specified on an evolving surface. In fact, however, this possibility was never used. Almost all coordinate systems for the Minkowski space-time are stationary, because they contain Lorentzian time t as one of coordinates. This coordinate is found in the standard Cartesian coordinates and almost all the rest curvilinear coordinate systems for the Minkowski space-time are built by replacing three spatial Cartesian coordinates in each 3-plane $t = \text{const}$ with curved ones. In our work [**13**] we have constructed an analogue to the triaxial system for the Minkowski space-time with one of axes being time-like.

One of simplest forms of this coordinate system, which represents an analogue to spherical coordinates, contains a light cone as one of coordinate surfaces which is labeled as, say, $u = 0$, where u is one of coordinates. Then the whole of interior of the cone is given by an inequality, say, $u > 0$. If time and place of polarization of a point-like particle is the vortex of the cone, the field produced by it is non-zero only under $u > 0$. In such a coordinate system the boundary conditions read that the field is zero on the boundary $u = 0$ and the field equation is to be solved in the domain $u > 0$. In the next section such a coordinate system will be presented.

4. The coordinate system of expanding world

A coordinate system with a light cone as one of coordinate surfaces, obtained from the standard spherical system $\{t, r, \theta, \varphi\}$ by the following transformation:

$$
\begin{aligned}
u &= \sqrt{c^2t^2 - r^2}, \qquad & v &= \operatorname{arctanh}\frac{r}{ct}, \\
ct &= u\cosh v, \qquad & r &= u\sinh v,
\end{aligned}
\tag{4.1}
$$

is shown on the Fig.1. In this system the coordinate surfaces $u = \text{const}$ are two-sheet hyperboloids $\sqrt{c^2t^2 - r^2} = \text{const}$ of which only one sheet $(t > 0)$ is taken, and surfaces $v = \text{const}$ are cones $r = \text{const} \cdot t$ so that both of these families contain the light cone $r = ct$ as the limiting cases $u = 0$ and $v = \infty$ correspondingly. The space taken in different moments of time forms the family of coordinate surfaces $u = \text{const}$ which represents a model of expanding world with u as the time coordinate. Therefore, there are two

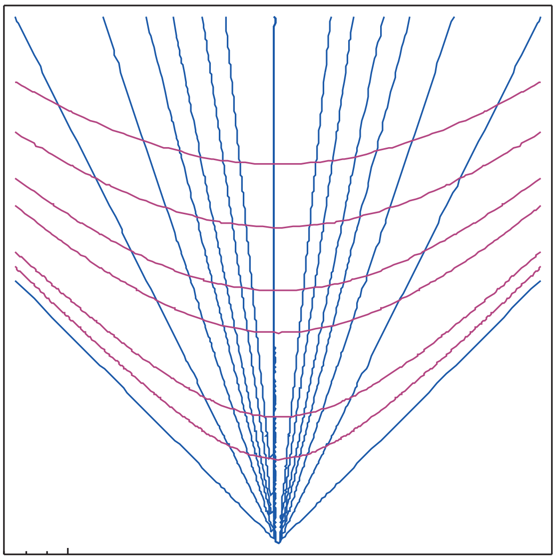

FIGURE 1. The coordinate system of expanding world in the Minkowski space-time.

ways a field can be made equal to zero on the light cone, to require that it vanishes under $u = 0$ disregard of its dependence on v and that it vanishes under $v = \infty$ disregard of its dependence on u. It is easier to require the latter because then in this case the field can be put independent on the coordinate u. So will be done below. Hereafter we put $c = 1$.

It is easy to make sure that this system is orthogonal because

$$\mathrm{d}u = \frac{t\,\mathrm{d}t - r\,\mathrm{d}r}{\sqrt{t^2 - r^2}}, \quad \mathrm{d}v = \frac{t\,\mathrm{d}r - r\,\mathrm{d}t}{t^2 - r^2} \tag{4.2}$$

and as the result, the scalar product of du and dv is zero. Below when deriving the field equation we use exterior differential forms and the following orthonormal frame:

$$\begin{gathered} \nu^0 = \mathrm{d}u, \qquad \nu^1 = u\,\mathrm{d}v, \\ \nu^2 = u \sinh v\,\mathrm{d}\theta, \quad \nu^3 = u \sinh v \sin\theta\,\mathrm{d}\varphi. \end{gathered} \tag{4.3}$$

Representation of the strengths 2-form in this frame simplifies application of the asterisk conjugation.

Consider a particle whose world line passes through the vortex of the light cone which in the standard spherical coordinates is given by $t = 0$, $r = 0$ and suppose that the particle is polarized at this point. Then the field is non-zero only inside the light cone out from this point, therefore, the coordinate system of expanding world is the most convenient one in this case. Since the fields of electric and magnetic dipoles are axially-symmetric, it is convenient to chose the coordinate system such that the axis $\sin\theta = 0$ is collinear with the dipole moment because in this case the field does not depend on the coordinate φ. It is convenient also to consider first the case of magnetic polarization then, if needed, obtain the strengths of electric dipole applying the asterisk operation to the strengths of magnetic dipole [**9**]. In case of magnetic polarization the vector potential which has only φ-component, and can be represented as a 1-form

$$\alpha = A(u, v, \theta)\,\mathrm{d}\varphi. \tag{4.4}$$

This 1-form satisfies the field equation [**9**]

$$\mathrm{d}\,{}^*\mathrm{d}\alpha = 0 \tag{4.5}$$

everywhere but the world line of the particle $v = 0$.

To obtain the explicit form of this equation we differentiate the 1-form α:

$$\begin{aligned}\mathrm{d}\alpha &= \frac{\partial A}{\partial u}\,\mathrm{d}u\wedge\mathrm{d}\varphi - \frac{\partial A}{\partial v}\,\mathrm{d}\varphi\wedge\mathrm{d}v + \frac{\partial A}{\partial\theta}\,\mathrm{d}\theta\wedge\mathrm{d}\varphi = \\ &= \frac{1}{u^2\sinh v\sin\theta}\left(u\frac{\partial A}{\partial u}\nu^0\wedge\nu^3 - \frac{\partial A}{\partial v}\,\nu^3\wedge\nu^1 + \frac{1}{\sinh v}\frac{\partial A}{\partial\theta}\,\nu^2\wedge\nu^3\right)\end{aligned} \tag{4.6}$$

where the definitions (4.3) have been used. Now, the asterisk conjugation yields

$$\begin{aligned}{}^*\mathrm{d}\alpha &= \frac{1}{u^2\sinh v\sin\theta}\left(-u\frac{\partial A}{\partial u}\nu^1\wedge\nu^2 - \frac{\partial A}{\partial v}\,\nu^0\wedge\nu^2 + \frac{1}{\sinh v}\,\frac{\partial A}{\partial\theta}\,\nu^0\wedge\nu^1\right) = \\ &= -\frac{u}{\sin\theta}\frac{\partial A}{\partial u}\,\mathrm{d}v\wedge\mathrm{d}\theta + \frac{1}{u\sin\theta}\left(-\frac{\partial A}{\partial v}\,\mathrm{d}u\wedge\mathrm{d}\theta + \frac{1}{\sinh^2 v}\frac{\partial A}{\partial\theta}\,\mathrm{d}u\wedge\mathrm{d}v\right)\end{aligned}$$

and it remains to take the exterior derivative of this 2-form. The differentiation yields

$$\begin{aligned}\mathrm{d}\,{}^*\mathrm{d}\alpha = &\left\{-\frac{1}{\sin\theta}\frac{\partial}{\partial u}\left(u\frac{\partial A}{\partial u}\right) + \right. \\ &\left. + \frac{1}{u\sin\theta}\left[\frac{\partial^2 A}{\partial v^2} + \frac{\sin\theta}{\sinh^2 v}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial A}{\partial\theta}\right)\right]\right\}\mathrm{d}u\wedge\mathrm{d}v\wedge\mathrm{d}\theta\end{aligned}$$

and since, due to the equation (4.5), this 3-form is equal to zero, we obtain the explicit form of the differential equation for the function $A(u, v, \theta)$:

$$-u\frac{\partial}{\partial u}\left(u\frac{\partial A}{\partial u}\right)+\frac{\partial^2 A}{\partial v^2}+\frac{\sin\theta}{\sinh^2 v}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial A}{\partial\theta}\right)=0. \tag{4.7}$$

This equation will be solved in the next section.

5. Solution of the field equation

The equation (4.7) separates and decays into three ordinary differential equations by the following substitution

$$A(u, v, \theta) = U(u)V(v)\Theta(\theta) \tag{5.1}$$

which after dividing by A takes the form

$$\frac{u}{U}\frac{\partial}{\partial u}\left(u\frac{\partial A}{\partial u}\right)=\frac{1}{V}\frac{\mathrm{d}^2 V}{\mathrm{d}v^2}+\frac{1}{\Theta\sinh^2 v}\sin\theta\,\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{\sin\theta}\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right).$$

Since opposite sides of the equation depend on different variables, each of them can only be constant. Put this constant equal to n^2. Then, the equation decays into the following ordinary differential equations

$$u\frac{\mathrm{d}}{\mathrm{d}u}\left(u\frac{\mathrm{d}U}{\mathrm{d}u}\right)=n^2U$$

$$\left(\frac{\sinh^2 v}{V}-n^2\sinh^2 v\right)=-\frac{\mathrm{d}^2 V}{\mathrm{d}v^2}+\frac{1}{\Theta}\sin\theta\,\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{\sin\theta}\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right).$$

Again, opposite sides of the second equation depend different variables, therefore both of them are constant, we denote it k and obtain the final form of ordinary differential equations for the single variable functions:

$$u\frac{\mathrm{d}}{\mathrm{d}u}\left(u\frac{\mathrm{d}U}{\mathrm{d}u}\right)=n^2U,$$

$$\sinh^2 v\,\frac{\mathrm{d}^2 V}{\mathrm{d}v^2}-(k+n^2\sinh^2 v)V=0,$$

$$\sin\theta\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{\sin\theta}\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right)+k\Theta=0$$

where n and k are separation constants. These three equations will be solved separately.

Evidently, solution of the first equation is $U(u) = u^n$, where we assume that some unit of length is introduced so that the time coordinate u is dimensionless. The field produced by an instantaneously polarized particle does not contain all particular solutions obtained this way. For example, the factor $\Theta(\theta)$ is a certain function which corresponds to the commonplace dipole field as the field in question will be in far future. Besides, the future $u \to \infty$ specifies a certain function $U(u)$. Indeed, magnetic strength produced by a dipole is known to descend as power -3 of distance, hence, natural components of the magnetic strength of the field in question are proportional to u^{-3}. As seen from the equation (4.6), v- and θ-derivatives of the function A have the common factor u^{-1}. Consequently, the function $U(u)$ is defined uniquely as

$$U(u) = \frac{1}{u}$$

that fixes the value of the constant $n = -1$ and specifies the explicit form of two other equations. as a result, they take the form

$$\frac{\mathrm{d}^2 V}{\mathrm{d}v^2} - V - \frac{kV}{\sinh^2 v} = 0,$$
$$\sin\theta \frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{\sin\theta}\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) + k\Theta = 0$$

The second equation can be solved by the substitution

$$f = \frac{1}{\sin\theta}\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}$$

which transforms it into the well-known equation

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}f}{\mathrm{d}\theta}\right) + kf = 0$$

whose solutions are Legendre polynomials $P_l(\cos\theta)$ with $k = l(l+1)$ [**3**]. Then the value $l = 1$ is the most relevant because in this case the v-component of the strength which is proportional to $A_\theta \sin\theta$, depends on θ as $\cos\theta$ that corresponds to the field of a dipole. Thus, $f = \cos\theta$ and for the function $\Theta(\theta)$ we obtain

$$\frac{\mathrm{d}\Theta}{\mathrm{d}\theta} = \frac{1}{2}\sin 2\theta, \quad \Theta(\theta) = \frac{1}{4}\sin^2\theta.$$

Since the value of the constant k is fixed as $k = 2$, the first equation takes the form

$$\frac{\mathrm{d}^2 V}{\mathrm{d}v^2} - V - \frac{2V}{\sinh^2 v} = 0. \tag{5.2}$$

To solve this equation we make the following substitution:

$$V = y(v)\sqrt{\sinh v}$$

and, first of all, calculate the first and second derivatives of the function $V(v)$ represented this way:

$$V' = \ y' \sinh^{1/2} v + \frac{y \cosh v}{2 \sinh^{1/2} v}$$
$$V'' = \ y'' \sinh^{1/2} v - \frac{y' \cosh v}{\sinh^{1/2} v} + \frac{y}{2}\left(1 - \frac{1}{\sinh^2 v}\right) \sinh^{1/2} v.$$

Substituting this yields the following equation for the function $y(u)$:

$$y'' + y' \coth u - \frac{y}{2}\left(1 + \frac{3}{\sinh^2 v}\right) = 0.$$

Solutions of this equation are Legendre functions of imaginary argument: $q_\mu^\lambda(\cosh u)$, where

$$\lambda = \frac{\sqrt{3} - 1}{2}, \quad \mu = \sqrt{\frac{3}{2}},$$

which vanishes under $v = \infty$. This gives explicit form of the function $V(v)$:

$$V(v) = q_\mu^\lambda(\sinh v)\sqrt{\sinh v}$$

and it remains to return to the φ-component of the vector potential A via the equation (5.1):

$$A(u, v) = \frac{q_\mu^\lambda(\sinh v)\sqrt{\sinh v}}{u} \sin^2 \theta$$

where a constant normalizing coefficient is omitted. Note that unlike the solution discussed above, this solution is in full agreement with the causality principle. This fact signifies that it is not sufficient to specify boundary conditions to obtain unique of solution of Maxwell equations, whereas addition of the causality principle, at least, reduces variety of solutions.

6. The field of an instantly polarized particle

Solution of the field equation (4.5) for the vector potential given by the 1-form (4.4) describes the field of a particle which was magnetized at a given moment of time. Exterior derivative of this 1-form (4.6) provides all components of electric and magnetic strengths produces by the particle. In the frame of expanding world (4.3) the field is represented by toroidal electric and poloidal magnetic strength with components

$$
\begin{aligned}
E_3 &= \frac{q_\mu^\lambda(\sinh v)\sqrt{\sinh v}}{u^3 \sinh v \sin\theta}, \\
H_1 &= -\frac{2\cos\theta}{u^3 \sinh v}\, q_\mu^\lambda(\cosh v), \\
H_2 &= -\frac{\sin\theta}{u^3 \sinh v}\, \frac{\mathrm{d}}{\mathrm{d}v}\left(q_\mu^\lambda(\cosh v)\sqrt{\sinh v}\right).
\end{aligned} \tag{6.1}
$$

These expressions allow to calculate the Poynting vector and its flux from the particle in the frame of expanding world. The expressions become more complicated if the strengths are referred to a Lorentzian frame by means of the equations (4.2), therefore we do not try to analyze them as they stand and first do it in the small neighborhood of the particle.

For this end, we return to the equation (5.2) and show that its solutions describe the field of a dipole which was “switched on” at the moment of time $u = 0$. Note that in a small enough neighborhood of the particle the value of the radial coordinate v is small and the equation can be approximately represented as

$$
\frac{\mathrm{d}^2 V}{\mathrm{d}v^2} - V - \frac{2V}{v^2} = 0.
$$

This is a particular case of the Whittaker equation whose solution is known [**19**], however, at the moment we do not need the exact solution. Similarly to the function $U(u)$ found above, this one is known to be approximately $V \approx v^{-1}$, and all we need is to make sure that this function really satisfies the equation under small enough values of v. Substituting this into the equation allows to neglect the term V in the domain and obtain the equation

$$
\frac{\mathrm{d}^2 V}{\mathrm{d}v^2} - \frac{2V}{v^2} = 0
$$

and see that the function under consideration satisfies it.

Hence, in a small enough neighborhood of the particle ($r \ll t$) the following approximation is in force:

$$A \approx \frac{\sin^2\theta}{uv}, \quad u = \sqrt{t^2 - r^2} \approx t, \quad v \approx \frac{r}{t}.$$

Substituting these equalities into the equation (4.6) yields the following approximate expression for the 2-form $\mathrm{d}\alpha$:

$$\mathrm{d}\alpha \approx \frac{\sin^2\theta}{r^2}\,\mathrm{d}\varphi \wedge \mathrm{d}r + \frac{\sin 2\theta}{r}\,\mathrm{d}\theta \wedge \mathrm{d}\varphi$$

which does not contain the electric strength and represents only the stationary strength produced by a dipole. Indeed, asterisk conjugation of this 2-form is

$$^*\,\mathrm{d}\alpha = \frac{\sin\theta}{r^2}\,\mathrm{d}t \wedge \mathrm{d}\theta + \frac{2\cos\theta}{r^3}\,\mathrm{d}t \wedge \mathrm{d}r$$

and its exterior derivative is identically zero. This result is quite expectable, because if at the moment of polarization the particle emitted electromagnetic waves, after a while they have propagated far away and cannot be detected near the particle. The radiation emitted can be detected near the light cone. To find it, we return to the equation (5.2) and consider it under another approximation.

The light cone out from the point and the moment of polarization coincides with coordinate surfaces $u = 0$ and $v = \infty$. Near this surface $r \approx t$ and the coordinate v has very high values. Therefore in the equation (5.2) the last term can be neglected and the equation takes the form

$$\frac{\mathrm{d}^2V}{\mathrm{d}v^2} - V = 0.$$

Since v is the radial coordinate, we choose the descending solution, hence, take the asymptotical solution $V = e^{-v}$. To express it in standard spherical coordinates, we use one of equations (4.1):

$$\tanh v = \frac{r}{t} \Rightarrow e^{-v} \approx \frac{\sqrt{t^2 - r^2}}{t}.$$

Then, an approximate form of the function A has no singularity on the light cone:

$$A = \frac{e^{-v}\sin^2\theta}{u} \approx \frac{\sin^2\theta}{t}, \quad \alpha \approx \frac{\sin^2\theta}{t}\,\mathrm{d}\varphi.$$

Differentiating it and substituting the result into the equation (4.6) yields

$$\mathrm{d}\alpha = -\frac{\sin^2\theta}{t^2}\,\mathrm{d}t\wedge\mathrm{d}\varphi + \frac{\sin 2\theta}{t}\,\mathrm{d}\theta\wedge\mathrm{d}\varphi. \tag{6.2}$$

Both electric and magnetic strengths are non-zero both of them depend on time, this part of the field contains radiation. To see that the 1-form α satisfies the field equation (4.5), take asterisk conjugation of its exterior derivative:

$$^*\,\mathrm{d}\alpha = \frac{\sin\theta}{t^2}\,\mathrm{d}r\wedge\mathrm{d}\theta + \frac{2\cos\theta}{tr^2}\,\mathrm{d}t\wedge\mathrm{d}r.$$

Straightforward checking out shows that this 2-form is approximately closed:

$$\mathrm{d}\,^*\mathrm{d}\alpha = -\frac{2\sin\theta}{t^3}\,\mathrm{d}t\wedge\mathrm{d}r\wedge\mathrm{d}\theta + \frac{2\sin\theta}{tr^2}\,\mathrm{d}t\wedge\mathrm{d}r\wedge\mathrm{d}\theta = \\ = -2\,\frac{t^2-r^2}{t^3r^2}\sin\theta\,\mathrm{d}t\wedge\mathrm{d}r\wedge\mathrm{d}\theta \approx 0$$

because the factor $t^2 - r^2$ is close to zero. The 2-form $\mathrm{d}\alpha$ (6.2) provides the following components of the electric and magnetic strengths:

$$E_\varphi = -\frac{\sin^2\theta}{t}, \quad H_r = \frac{\sin 2\theta}{tr^2}.$$

Both strengths descend as t^{-1} and contrary to expectations, the Poynting vector has only the θ-component. This unexpected result demonstrates deficiency of the coordinate system used, which possesses a singularity on the light cone. Purely radial magnetic strength on the light cone displays a non-zero surface density of magnetic charge on the sphere which expands with speed of light.

7. Causality and dipole radiation

Electromagnetic field of a dipole whose moment behaves as an arbitrary function of time, contains a purely radiative part called dipole radiation. Since Maxwell equations are linear, it is generally adopted that to obtain the field of dipole whose moment is given by an arbitrary function of time, it suffices to know the field in the case of sinusoidally oscillating dipole moment and then compose the general solution in the form of Fourier expansion. However, this approach does not allow to select solutions which obey the relativistic causality principle. The case of instant polarization of a particle is quite special in this sense. Since change of the moment occurs only in one space-time point, it is possible to construct a special coordinate system with this point as the origin, in which any solution of the Maxwell equations satisfies the principle by construction. So is solution for the case of polarized particle (6.1) obtained in coordinates of expanding world (4.1).

In this coordinate system the coordinate u is exactly the proper time on a coordinate line $v = \text{const}$, $\theta = \text{const}$. Therefore, if dipole moment of a particle suspended in the origin of coordinated $v = 0$ is given by an arbitrary function of time $f(t)$, this function can be replaced by the function $f(u)$. Maxwell equations separate in these coordinates, therefore complete a formal solution, which is zero beyond the light cone, can be built for the field of dipole with moment specified this way. However, in general, this solution does not obey the relativistic causality principle. The case of single change of the dipole moment is an exception because the space-time point of this event is used as the vortex of the light cone. If the dipole moment changes as a continuous function, each point of its world line should be used in this capacity. The corresponding coordinate system is known and can be built of the standard spherical coordinates by the following coordinate transformations:

$$u = \frac{t + r}{\sqrt{2}}, \quad v = \frac{t - r}{\sqrt{2}}.$$

The separation procedure in this system consists in replacing the new coordinates with old ones that is the same as to solve the equations in the spherical coordinates. Therefore this system does not provide the desired solution of the Maxwell equations which manifestly satisfies the relativistic causality principle.

The solution derived in the previous section allows to obtain the field of a dipole moment given by a piecewise-constant function of time. If the moment behaves this way, the world line of the dipole can be decomposed into segments within which the moment is constant and all changes occur only in their endpoints. Each change produces the field of polarized particle and the entire field is sum of these contributions. Evidently, each junction of segments appears as the vortex of a light cone inside which its contribution is confined. It is convenient to refer each contribution to a standard system of spherical coordinates $\{t, r, \theta, \varphi\}$ in which the sum of all contributions can be found. Further development of this techniques might allow to pass to the limit of infinitesimal segments and turn the sum into an integral. As a result, the field of a dipole whose moment behaves as a continuous function of time, which obeys the relativistic causality principle, can be obtained.

CHAPTER 3

Causality in celestial mechanics

1. On experimental and ideal foundations of physics

Physics is believed to be built on a rigorous experimental basis. So should be mechanics as its part. Conservation laws play the basic role in the foundations of classical mechanics, therefore, they should be established experimentally before the main equations of this theory appeared. However, no experiments are known in which these laws have been discovered. In fact, no laws could be discovered from experiment alone until someone already knows what kind of law he wants to see in it. To see it, one must know the law and needs the experiment only to confirm his belief. It will be shown below the main laws of classical mechanics have been discovered without any experiments at all, in fact, they have been derived from the Newtonian equations of motion. The generally accepted ideology which claims that physics is a science based first of all, on experimental data, is in an apparent contradiction with these facts. In this section we consider the role of astronomy in foundations of mechanics and particularly, in discovery of conservation laws.

The main laws in classical mechanics are energy, momentum and angular momentum conservation laws. One of them exists in two forms, but can be observed only in one of them. Another could well be discovered experimentally, but this was not happen until it was derived mathematically along with the third one. Below we analyze these discoveries starting from the momentum conservation law.

This law exists in two distinct forms, one of which is observed in each collision of two bodies and another, which actually requires that the space is manifestly flat and which, naturally, have never been observed in an experiment. In fact, this law is nothing but Galileo's or Descartes' guesswork. The law of angular momentum conservation has much more recent origin. It was discovered only as a consequence of equations of motion, hence, this happened when these equations were known. The most interesting case is discovery of the energy conservation law.

One experiment in which this law could well be discovered, was completed many times and its results exposed the energy conservation law in numerous different forms, but nevertheless, the law was not discovered this way. This experiment consists of numerous attempts to implement perpetuum motion which lasted for centuries and finished only before the World War **I** [**20**], when the energy conservation law was well-known. Though each of constructions of perpetuum mobile could well be interpreted as a demonstration of the law, the law itself was not deduced from these evidences. In fact, conservation laws were derived when trying to solve the Newtonian equations of motion and appeared first in the form of first integrals of motion. These derivations will be completed in the next section.

Search for the very basic foundations of physics leads to astronomy and celestial mechanics. Integrals of motion have been derived from Newtonian equations of motion, which were composed on the basis of Kepler's laws, Galileo's ideas and experiments and Descartes' philosophy. One more discovery made in medieval astronomy was speed of light and retardation caused by its finiteness. In spite of evident importance of this discovery, it was ignored because it makes retardation an obligatory detail of almost any astronomical observation that destroys the whole of Newton's theory.

Any observer refers all events in the Universe to his own frame and his own chronology which are based on the notions of the instant space as he imagines it and his proper time. In his chronology every event happened at a distance r from him is seen with retardation equal to r/c. Retardation is a fundamental phenomenon, which exposes properties of space and time, therefore it does not depend on the any material conditions. It can depend on choice of frame or place in the space, but it takes place for any two points in the space. A principle which claims this, could well be accepted as one of fundamental principles of physics that would destroy Newtonian theory before it was created. Since this principle was incompatible with the Newtonian theory, it was not formulated and taken into account. This example shows that not every experimental fact can be used when building a theory, some of facts need to be ignored to avoid crush of the whole construction before it is completed.

Nowadays there is no danger that Newtonian theory will be destroyed by taking this fundamental principle into account. Presently it is quite safe to return to the state when Newtonian theory was under construction and

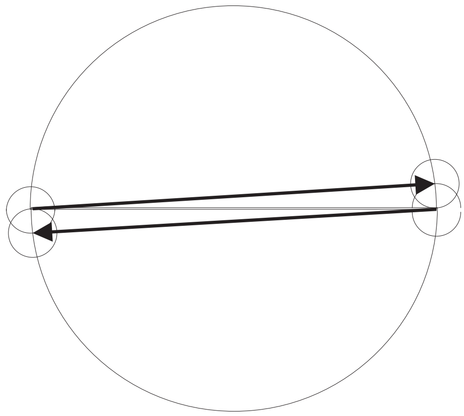

FIGURE 1. Retardation of force in a two-body system with circular orbits.

look what kind of laws would be discovered if the principle of retardation of everything was not ignored. If retardation is a general law, it must reveal also in gravitational forces between the bodies that may well change laws and even definitions used in celestial mechanics.

Electromagnetic waves play no role in celestial mechanics, they allow astronomers to observe motion of celestial bodies in the sky, but do not affect it. Unlike electromagnetic waves, gravitational forces predetermine the laws of this motion. Therefore, if motion of celestial bodies obeys the causality principle, they are gravitational forces which should obey it first of all. Consequently, the causality principle requires that each body moves in retarded gravitational fields of other bodies. In other words, equations of motion of each particle contains positions of all the rest particles taken with retardation. Retarded forces in a two-body with circular orbits system are shown on the Fig.1. As seen from the picture, retarded forces form a torque which would spin up the system. This fact signifies that the problem of retardation of gravitational force in celestial mechanics is important and in this chapter we try to introduce retardation into solutions of the simplest problems of the subject. It turns out that the main problem is to combine retardation with conservation laws. Therefore below we start with discussing conservation laws as consequences of the equations of motion.

2. Conservation laws in celestial and classical mechanics

It was noticed by E. Mach that all laws of mechanics have been discovered, first of all, thanks to existence of the system of immobile stars [**21**]. Further development of mechanics of a mass point started actually from